Examining Computational and Conceptual Models for Seasonal Carbon Fluctuation

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The carbon cycle is a key component of the Earth's climate, the main function of the carbon cycle is to moderate global temperatures by absorbing infrared radiation. The carbon cycle specifically refers to carbon recycling on Earth; however, while carbon is present in compounds in organisms and abiotic substances in various forms, the carbon cycle is driven through the exchange and sequestration of carbon dioxide (CO_2) through various intermediaries. A significant property of the carbon cycle is that it can be examined through two time scales, a long-term carbon cycle that operates over millions of years and a shorter time scale in which we can see the cycle operate over a time scale of a few hundred years. However, over the past 100 years, as humans have intensified climate change, we have seen changes in carbon cycle dynamics. This paper will examine factors driving the short-term carbon cycle and evaluate models that attempt to project short-term carbon cycle dynamics into the future.

Distinguishing processes in the long-term from the short-term cycle is important in understanding the underlying causes of carbon fluctuations. The long-term carbon cycle operates by forming organic carbon from dead life forms; these carbon stores are created on a time scale of approximately 300 million years. Carbon is released through volcanic eruptions and the natural combustion of fossil fuels. On a long time scale, carbon from the atmosphere is absorbed into the ocean, where it forms rock-limestone ($\mathrm{CaCO_3}$) and dolomite ($\mathrm{CaMg(CO_3)_2}$) (Berner 2003). The formation of these rocks is especially significant, as on a long time scale, the carbon cycle can drive the formation of new types of rock. Evidence of the long-term carbon cycle has been observed by collecting ice cores from Antarctica, which provide historical samples of atmospheric compositions over the past 800,000 years (Berner 2003. We can observe long-term fluctuations in carbon dioxide through ice cores, which correspond with the warming and cooling of the globe. We see high average global temperatures at high concentrations of atmospheric carbon dioxide, while at lower concentrations, ice cores provide data signaling lower global temperatures.

On the time-scale of human lifetimes, we typically do not see the consequences of the long-term carbon cycle. The majority of the fluctuations we notice are evidenced through the short-term carbon cycle, which is the data primarily obtained in atmospheric carbon readings measured by climate scientists. The short-term climate cycle is primarily driven by vegetation; photosynthetic processes temporarily sequester carbon stored as glucose; however, this carbon is later released through respiration processes that release carbon dioxide into the atmosphere. While they may not play as significant a role, dynamics between the ocean and the atmosphere also impact the amount of carbon left in the atmosphere. Carbon is temporarily sequestered in the lower levels of the ocean, as layers in the ocean contract at colder temperatures; however, this carbon is released back into the atmosphere during warmer months, as ocean water evaporates and reenters the atmosphere. Before the Anthropocene, since the short-term carbon cycle functioned through these three processes; we saw smaller increases in overall carbon levels during that time. However, technological developments caused humans to begin burning fossil fuels, thus releasing extra carbon into the atmosphere. This additional source of carbon dioxide emissions, while still moderated through oceanic sequestration, has led to significant changes to the

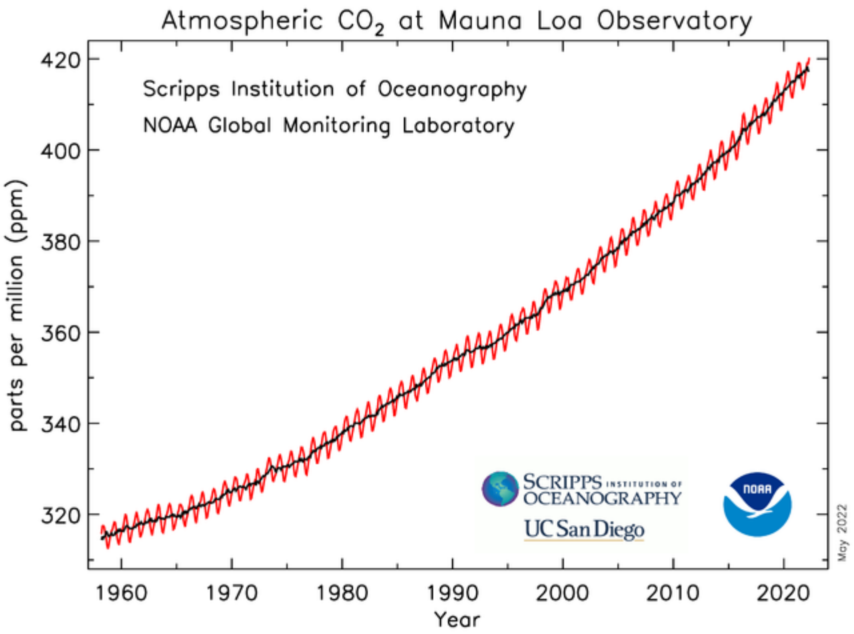

Figure 1: Atmospheric Carbon Data from Mauna Loa Observatory

dynamics of the short term carbon cycle, where we no longer see steady carbon levels on a year to year basis and instead are observing a steady increase in atmospheric carbon levels.

We can then understand seasonal carbon fluctuations by attributing them to the dynamics of the short-term carbon cycle. When looking at annual atmospheric carbon levels, we notice a monotonic increase in atmospheric carbon levels in the last 50 years. However, when looking at carbon levels from year to year, we notice that atmospheric carbon levels fluctuate cyclically within a single year. For example, at higher latitudes, including the United States and Eurasia, atmospheric carbon levels reach maxima during the winter months while they reach minima during the winter months. Seasonal fluctuations in carbon dioxide levels are significant to climate scientists. The cyclical patterns of carbon dioxide concentrations provide important information on how the ocean and the biosphere interact with the atmosphere. Furthermore, seasonal carbon fluctuations are important in understanding the incremental changes caused by increased greenhouse gas emissions. Over a longer time scale of 20 years, it is evident that fossil fuel emissions have caused an increase in carbon dioxide in the atmosphere. However, we can gain a far more detailed understanding by examining the effects of fossil fuel emissions on a seasonal climate scale. To quantify the seasonal fluctuation in carbon dioxide over a year, scientists take the seasonal carbon amplitude (SCA), which measures the difference between the maximum and minimum carbon values during a year. To understand how climate change affects our climate on an extremely short time scale, scientists have attempted to model changes in seasonal carbon amplitude over time.

The most extensive measurement of seasonal carbon fluctuations is recorded at the Mauna Loa Observatory in Hawaii, which has daily measurements of atmospheric carbon dioxide levels from 1960 to the present, as seen below. The simplest model for carbon amplitudes proposes a linear relationship between current carbon dioxide levels using a regression model applied to the data from the Mauna Loa Observatory and Point Barrow in Alaska and simulated data at various other latitudes. By averaging the regression coefficients from the various observation sites, the authors proposed a model that suggested a direct relationship between an increase in carbon dioxide levels ($\Delta C_A(t)$) and seasonal carbon amplitude (Wenzel et al. 2016).

$$\Delta C_{A,\mathrm{ampl}}(t) = -2.54 + 0.050\Delta C_A(t)$$

On a fundamental level, the model makes sense, as higher atmospheric carbon levels lead to increased vegetation, leading to higher seasonal carbon levels. However, empirical data observations have shown that the model proposed does not always predict accurately. Daily measurements at Mauna Loa in Hawaii have shown that there has been a steady increase in overall carbon dioxide levels in the past 60 years. However, seasonal carbon amplitudes have not necessarily followed the same trend. Calculations of seasonal carbon amplitudes show observed declines in SCA. In particular, from 1998 to 2003, there was a noted decrease in carbon dioxide, which was attributed to a decrease in vegetation caused by drought (Buermann et al. 2007). However, as land temperatures returned to normal and drought conditions subsided, an increase in amplitude was noted.

As seen in the trends from 1998 to 2003, vegetation is among the factors which cause changes in atmospheric carbon levels throughout the year. Vegetation contributes to seasonal carbon fluctuation through two main processes, photosynthesis and respiration. During photosynthesis, plants generate energy by absorbing carbon dioxide from the atmosphere and converting it into glucose. By storing glucose in their roots, plants end up sequestering carbon in their roots. However, these stores are not long-term sequesters of carbon. Plants reintroduce carbon dioxide into the atmosphere through respiration by breaking down glucose to convert it into energy, carbon dioxide, and water. Energy-generating processes in plants are a significant contributor to seasonal carbon fluctuations, as the overall rates of photosynthesis and respiration vary through the seasons. As temperature levels increase, we see an increase in photosynthetic activity. This is backed up by observational data on photosynthetic rates measured through the rate of electron transport, where photosynthetic activity peaks at leaf temperatures between 25 and 35 degrees Celsius (Lin et al. 2013). However, during colder months, plants rely more on respiration for energy generation. Consequently, we see that during the warmer months of the summer and spring in the United States and Eurasia, we see local minima in atmospheric carbon levels; meanwhile, during the fall and winter, we observe local maxima in atmospheric levels.

Given that vegetation is a driver of atmospheric carbon levels in a year, changes in the amount of vegetation can drastically impact the seasonal carbon amplitudes over a year. Large amounts of vegetation cause increased seasonal carbon amplitudes as more carbon is sequestered through photosynthetic processes and released through respiration. Therefore, during years with higher vegetation, we observe that both the maximum and minimum values of carbon dioxide increase and decrease, respectively, thus leading to an overall greater seasonal carbon amplitude. Conversely, during years with lower vegetation, there is a decrease in seasonal carbon amplitude as we have large minimum values of carbon dioxide and lower maximum values of carbon dioxide and a lower seasonal carbon amplitude.

The amount of vegetation present at a given time allows scientists to better understand seasonal carbon fluctuations. A major consideration in modeling seasonal carbon fluctuations is factors that affect the amount of vegetation present during a year. Among the most significant factors is temperature; in countries with higher temperatures, the growing season tends to be longer, as plants are alive for longer periods of time during a year, which allows them to photosynthesize for longer periods of time. Temperature is significant as it is the driving factor for higher latitudes that typically have temperate climates; during years with higher average temperatures, we see that the magnitude of seasonal carbon amplitude is greater, while during years with lower average temperatures, leading to lower seasonal carbon amplitude values (Anoruo and Anoruo 2021).

An additional factor that affects the amount of vegetation is rainfall. During periods of higher rainfall, plants can grow more and undergo greater levels of photosynthesis, thus sequestering more carbon dioxide into the atmosphere. In countries such as India, where temperatures are at steady levels optimal for photosynthetic activity, we still observe seasonal fluctuations. The fluctuations in India are characterized by their 2-season cycle, with a monsoon season characterized by heavy rainfall, compared to the dry post-monsoon season (Metya et al. 2021). During the monsoon season, increased rainfall causes increased vegetation growth, as plants have more water to undergo photosynthesis, thus stimulating plant growth; therefore, during periods of heavy rainfall, we can observe through seasonal data that atmospheric carbon dioxide levels reach their minimum values during the monsoon (Metya et al. 2021).

Drought has the opposite effect of rainfall on vegetation growth, temperatures that are too high lead to inadequate water stores for vegetation, preventing plants from undergoing photosynthesis. Therefore, during drought conditions, vegetation growth decreases, causing a decline in seasonal carbon amplitude. The effects of drought were clearly seen during the 1998-2003 drought in North America, where data from the Mauna Loa Observatory demonstrated a decline in SCA (Buermann et al. 2007).

Finally, the creation of cropland through agricultural intensification has led to increases in vegetation and thus an increase in seasonal carbon amplitude. Significantly, increases in cropland from 1959 to 1984 have led to an increase of around 25-44% in the SCA trend from 1959-to 1984 (Wang et al. 2020). However, recently, the estimated effect of cropland on SCA has decreased as cropland intensification has reduced in magnitude. This is mainly because cropland, while still undergoing photosynthetic and respiratory processes, sequesters far less carbon than their larger tree and shrub counterparts, offering lower benefits than other vegetation sources and thus requiring far greater comparative growth in arable land to have a sizable impact on SCA.

Logically, given that increases in vegetation have such a strong correlation with seasonal atmospheric carbon levels, one would infer that one could effectively model carbon emissions by measuring the vegetation area over a year. However, studies measuring the correlation between NDVI (an index of the amount of land occupied by vegetation in a specific area) and seasonal carbon amplitudes found that there are exceptions to this correlation, making an approach predicting SCA solely using vegetation data unreliable.

Among the exceptions to the vegetation-SCA correlation are the seasonal carbon fluctuations measured in Nigeria. In Nigeria, we find that there are inconsistent correlations between vegetation and atmospheric carbon dioxide levels. From a period from 2003 to 2008, the authors found a weak positive correlation from 2003 to 2004 of 0.03; however, from 2005 to 2006, there was a stronger correlation of 0.73 (Anoruo and Anoruo 2021). However, during 2007-2008, the correlation between vegetation index and atmospheric levels flipped to -0.36, signaling a negative correlation between vegetation and atmospheric levels. Nigeria's data stands as an outlier in comparison to other countries in Africa, such as Mauritania and Sudan, which measure consistently steady relationships between carbon dioxide and vegetation. The study's authors conclude that the inconsistent data measured in Nigeria is due to increased anthropogenic carbon emissions, which serve as an additional source of carbon dioxide for the atmosphere (Anoruo and Anoruo 2021). Furthermore, anthropogenic carbon emissions lead to low greening, causing seasonal carbon levels to be driven primarily by fluctuations in human carbon emissions instead of photosynthesis and vegetation.

Additionally, in less industrialized countries such as Sudan and Mauritania, where anthropogenic emissions play less of a role, we see an increased and more steady relationship between vegetation and atmospheric carbon roles. Trends of anthropogenic carbon forcing apply to other rapidly industrialized countries. Researchers observed an even more extreme trend in China than in Nigeria; from 2003 to 2008, the recorded correlation between NDVI and carbon dioxide ranged from 0.12 to -0.22, signaling a very low correlation between vegetation and carbon dioxide in China (Anoruo and Anoruo 2021). The low correlation is explained by the fact that 30% of carbon dioxide concentrations are generated through industrial activities concentrated in larger cities; thus, large anthropogenic emissions often mask the effects of vegetation present in rural areas. Therefore, researchers recommend surface-based observation to better understand the influence of vegetation and other factors on China's atmospheric carbon levels (Anoruo and Anoruo 2021).

The findings on regional atmospheric carbon concentrations thus suggest a more robust model that can consider the effects of vegetation on seasonal carbon amplitudes and factors in anthropogenic carbon emissions. While a vegetation model calculating the net global photosynthetic and respiratory outputs would likely present accurate global trends for climate fluctuations; the model would be unable to return accurate numerical results for SCA, as the model excludes numerous other contributors of atmospheric carbon which have significant impact on atmospheric carbon levels. To improve the numerical accuracy, a model should also consider other carbon sequestration and release sources, such as ocean-atmospheric dynamics, to produce more accurate results. In addition, to gain accurate latitude predictions of atmospheric carbon levels, the model should include atmospheric dynamics such as wind flow and concentration gradients.

A potential model for seasonal carbon models was presented in 1986 by Dale A. Gillette and Elgene O. Box (Gillette and Box 1986). The model predicts atmospheric carbon fluctuations by calculating fluxes for systems that exchange carbon quickly. The model provides equations for air-sea exchanges, atmospheric transport, and fossil fuel emissions, all as fluxes that pass in and out of the atmosphere. Therefore the model in its simplest form is expressed as the solution to the conservation of mass equation, given that under a short time scale, the total carbon budget of the Earth remains the same from year-to-year expressed as

$$\rho \int \frac{\partial x}{\partial t} dV + \int (F \cdot \tilde{n}) dA - S_0 - S_B - S_A = 0$$

The variable x which is the mixing ratio of carbon in the atmosphere is expressed as the sum of the fluxes: S_0 corresponds to oceanic flux, S_B corresponds with biosphere flux, and S_A corresponds to fossil fuel emissions. Finally, to provide latitudinal dependencies for the model, $\int (F \cdot \tilde{n}) dA$ expresses the atmospheric transport of carbon. Using the solution to the differential equation above, the authors can calculate the change in atmospheric carbon with respect to time. The model uses one-day increments, allowing for the model to predict atmospheric carbon levels across an entire year, allowing for analysis of seasonal carbon fluctuations and predictions for future seasonal carbon amplitudes. The authors computed values for three years and ten-year spans using atmospheric and oceanic data from previous studies and found that their model showed fair agreement with observed carbon dioxide amplitudes.

The model's primary component consists of a Bioflux model. The model simplifies photosynthetic and respiratory rates through a quantity known as Gross Plant Productivity (GPP), which is derived from the net photosynthetic rate. The model can then express the net flux through the biosphere as GPP minus the respiration rate (R). Additionally, the model considers that plants eventually die out, which acts as a net emitter of carbon into the atmosphere. Therefore an additional term (D) is added to the climate balance model as the model assumes that all dead plants (litter) are eventually microbially decomposed.

In a separate paper written by Elgene O. Box, the process for computing the carbon balance is outlined. The value for the net flux is represented as the Net Primary Production (NPP). The model attributes increases in gross production to warmth and water availability which are both locally parametrized. NPP becomes larger in warmer and wetter locations such as the equator. However, in these regions, respiration also increases with higher temperatures, thus causing the NPP to be negligible in these locations, thus contributing very little to seasonal carbon fluctuations. As such, NPP has a far greater impact on carbon fluctuations in temperate climates further from the equator. Therefore the model calculates Gross Plant Productivity through geographic data where a vegetation index (NDVI) is established to gain a baseline for primary production (Box, Holben, and Kalb 1989). The data was collected through primary production measurements at 130 field sites. These measurements were conducted using satellite measurement, where analysis of the pixel greenness allowed the authors to calculate the portion of land occupied by vegetation. However, Box points out that the number of sites observed is insufficient for calculating global plant productivity; so additional data for 1596 sites was collected for monthly temperature and precipitation values.

In addition to using the values from Gross Plant Productivity from empirical research, the Box and Gillette model fully localizes gross primary production data by adding additional data to the model by solving for monthly values of GPP by using the precipitation and temperature data measured at additional data sites. At latitude values where Gross Plant Productivity was not empirically measured, the model utilized annual GPP values from an older study. In the case of bioflux, the authors relate the gross production for each month to the actual evapotranspiration rate (AET): the rate at which water is transferred from the land to the atmosphere through transpiration by plants, using temperature and precipitation data. This relationship makes sense biologically, as the rate of water use directly corresponds to the rate of water used during photosynthesis. Furthermore, the relationship between AET and vegetation is numerically shown in Box's paper, where we find that there is a direct exponential relationship between AET and the NDVI. In their subsequent paper, the authors reason that AET and GPP must then be directly related, given that NDVI and GPP are directly related. Therefore the GPP for a given month is given by the following equation

$$\mathrm{GPP}_i = \frac{\mathrm{AET}_i}{\mathrm{AET}_y}\mathrm{GPP}_y \tag{1}$$

where the monthly GPP is modeled as the fraction of the annual GPP that is the ratio of monthly AET and yearly AET.

When calculating the net carbon release, Box and Gillette used an additional model which similarly utilizes the actual evapotranspiration rate as an index for decomposition. In a study by Meentemeyer, the authors establish a model using linear regression. The relationship between AET and litter decay rates in a vegetated system was highly correlated with an r-value of .98. The high correlation was justified given that litter moisture availability is among the main driving factors for plant decomposition in nature. Therefore the paper by Meentemeyer contributes a multiple linear equation using annual AET and monthly AET as parameters for annual decomposition rates. Box and Gillette utilize the parameterization given by Meentemeyer to estimate bioflux by substituting the output of the linear equation for the production of litter term in the schematic equation for Net Primary Productivity, as shown below

$$Y_1 = -1.31369 + 0.05350X_1 + 0.18472X_2$$

where Y_1 is the annual weight lost, and X_1 and X_2 are respectivley parameters for annual and monthly evanspiration. However, given that the decomposition rate was given in annual rate, Box and Gillette modified the model to calculate monthly decomposition rates using partial decomposition rates or reducing the parameters for annual AET and current AET accordingly so that the decomposition value returned represents the decomposition for a given month. Finally, to account for respiration, Box and Gillette relate temperature to respiration rates. The model utilizes a Q_{10} value of 2, which implies that the rate of respiration increases by 2 sec^{-1} for every 10 degrees Celsius increase.

Once parameterizations for monthly production, respiration, and decomposition were determined, the authors applied the values to 1300 climate data sites worldwide to provide monthly biosphere fluxes at various latitudes. The sum of the three values is shown in the following equation where ΔC is the total biosphere flux.

$$\Delta C = (GPP - R_d) - D \tag{2}$$

The values for carbon flux at different latitudes were then smoothed to ensure steady-state conditions (no net annual intake or outtake of carbon due to biosphere fluxes) and to produce agreement across latitude bands.

The second significant source of carbon fluctuations is sequestration and release caused by ocean and atmosphere exchange. Box and Gillette assume that carbon exchange in the ocean occurs between two non-circulating bands of water, where the upper layer is referred to as the mixed layer. The model assumes that the depth of the mixed layer can be expressed through a cosine function with parameters for the day of year and latitude. The second lower layer interacts with the mixed layer due to seasonal heating and cooling. During heating, the mixed layer expands, and some of the water from the deeper layer is added to the mixed layer. During contraction, the deeper layer adds water from the mixed layer. This process is significant as dissolved carbon is incorporated into the lower layer and sequestered or moved up to the mixed layer, where water evaporates, and carbon is brought back up to the atmosphere. Box and Gillette model this process through two conservation of mass equations. The change in carbon concentration for the mixed layer is expressed as

$$dc_1 = (\frac{dh}{h})(c_2 - c_1)$$

where when the mixed layer expands $\frac{dh}{h}$, we see that dc_1 increases as water from the deeper layer are incorporated into the mixed layer. Similarly, for the second layer, the concentrations of carbon are modeled as

$$dc_2 = (\frac{dh}{D-h})(c_1 - c_2)$$

we can see when the mixed layer contracts $\frac{dh}{D-h}$ increases, and thus the concentration of carbon dioxide increases.

The Box and Gillette Model establishes the dynamics of carbon transport between layers; however, an additional process is required to establish flux between the ocean and atmosphere. The mass exchange of CO_2 is driven through the interface between the atmosphere and the mixed layer, where carbon is absorbed from the atmosphere during cooler temperatures through condensation, where it is sequestered into the ocean; during warmer months, carbon is released from the mixed layer where it is reincorporated into the atmosphere. Box and Gillette define the mass exchange of CO_2 as an Ohmic process driven by differences in partial pressure of CO_2 in the atmosphere in the atmosphere and partial pressure of CO_2 in the water. Ohmic functions follow the following relationship Electric Potential = Current * Resistance, typically seen in circuits. However, Box and Gillette present an ohmic relationship by representing Electric Potential as the gradient caused by the partial pressures in the ocean and the air. Similarly, the inverse of resistance is defined as transfer velocity, representing wind velocities accelerating ocean carbon transfers. The authors then represent carbon flux caused by ocean-air transport as current; one can approximate ocean-atmospheric flux by multiplying the partial pressure gradient by the transfer velocity as shown here

$$F = A[\rho\mathrm{CO}_2(\mathrm{ocean}) - \rho\mathrm{CO}_2(\mathrm{air})]V \tag{3}$$

Box and Gillette explicitly compute the partial pressure of CO_2 in the ocean by utilizing a previous study by Bacastow in equilibrium by solving the simultaneous solutions for equations regulating alkalinity, salinity, and total dissolved inorganic carbon (TDIC). This approach adds time and latitude dependencies to the model, given that alkalinity, salinity, and TDIC all vary based on latitude and time. The authors then rely on experimentally determined dissolution constants to calculate the concentration of carbon dioxide dissolved in the ocean, thus allowing them to calculate the associated partial pressure in the ocean. We can see the associated equation expressing the concentration of carbon as the sum of ocean carbon concentration in the different layers and dissolution constant K

$$[[\mathrm{CO}_2] = [\sum \mathrm{CO}_2] - \sqrt{K_1'[\sum \mathrm{CO}_2]} \tag{4}$$

To compute the partial pressure of CO_2 in the air, the authors assume that CO_2 concentrations in the air remain uniform and thus could be determined simply by using the percentage of CO_2 in the atmosphere.

To compute the transfer velocity, Box and Gillette relate that transfer velocity is proportional to friction velocity caused by carbon dioxide moving through the air and through a Schmidt's number of $\frac{2}{3}$, which represents the acceleration of CO_2 transfer due to wind speeds. As such, the authors can establish a time-dependent model for transfer velocity utilizing a phyisical constant u_* representing friction due to air and critical velocity, and a parameter for air temperature t which provides time dependencies, as shown in the following equation:

$$V = 0.048(1 + 0.007t)u_* \tag{5}$$

The results of plotting transfer velocities across a year show that at higher latitudes, we see lower transfer velocities between the summer months, while during the winter, we see higher transfer velocities. This trend makes sense, considering that carbon is sequestering the ocean during colder months in the northern hemisphere, while benign is released during warmer months. Finally, Box and Gillette then calculated the flux due to atmosphere-ocean exchange as the potential difference between CO_2 in the air and CO_2 in the ocean multiplied by the transfer velocity.

The Box and Gillette model also considers carbon transport due to wind and concentration gradients. Box and Gillette treat the atmosphere as a box with two layers, the stratosphere, and the troposphere. The model assumes that transport only occurs on-axis in the stratosphere; therefore, there is no horizontal transport in the stratosphere. Furthermore, the model attributes mixing in the troposphere to wind fields through the equator. The net direction of CO_2 travel in the atmosphere can be represented through another flux model, where the net flux is the solution of the integral

$$\int (\mathbf{F} \cdot \mathrm{n})\ dA = F_T A_T - F_B A_B - F_N A_N + F_S A_S \tag{6}$$

The authors then simplify the integral to only take into account the net fluxes vertically and horizontally, eliminating the need for four separate flux terms, since the model assumes that transport does not occur in longitudinal directions. Vertical Flux is represented as,

$$F_V = \{-\rho(z)K_z(z)(dx/dz)_a + \rho(z)[V(z, y) \cdot k]x\} \tag{7}$$

which increases as vertical wind speed increases and decreases as vertical diffusion increases. Similarly, Horizontal diffusion is represented as

$$F_H = \{-\rho(z)K, (y, z)(dx/dy)_y + \rho(z)[V(z, y) \cdot j]x\} \tag{8}$$

we see an increase in flux as horizontal wind speed increases and a decrease as vertical diffusion increases. The model derives values of wind speeds from observational data. Additionally, the model derives diffusion coefficients from the NOAA/Air Resources Laboratory simulations, allowing parameterization based on time and latitude. Through the use of horizontal and vertical fluxes, the model can predict the distributions of atmospheric carbon dioxide for a given day.

The final portion of the Box and Gillette model focuses on human emissions from fossil fuel emissions and limestone conversion. The model utilizes estimates from an outside source, Rotty et al., who measured carbon emissions in various different countries in different latitude bands. The Box and Gillette model incorporates data for latitude bands ranging 10 degrees in size and adds them as a net emitter of carbon to the model. Significantly, the model does not assume that fossil fuel emissions fluctuate yearly and instead keeps fossil fuel contributions constant across latitude bands. However, the model does argue that should fossil fuel emissions have been assumed to have seasonality, the results would have had slightly higher amplitudes, attributed to heightened fossil fuel emissions during colder months.

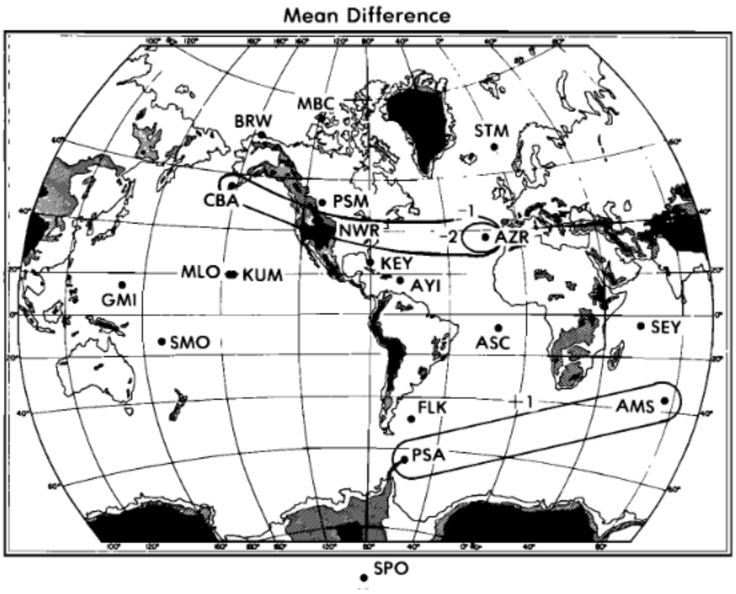

Figure 2: Mean Difference between Flask Data and Model

The overall model results are calculated using the original conservation of mass equation

$$\rho \int \frac{\partial x}{\partial t} dV + \int (F \cdot \tilde{n}) dA - S_0 - S_B - S_A = 0$$

and inserting the daily values for each of the associated fluxes. Box and Gillette carried out two runs of the model, one on a ten-year time scale and one on a 3-year time scale. The results of each run were then compared to experimental results collected by the NOAA Air Resources Laboratory through flask sampling of the atmosphere at different locations. The authors found that their model successfully predicted the large-scale features contributing to atmospheric CO_2 levels. When comparing their model to flask results, the mean difference between the observational data and the flask results was only 4ppm for 12 of the 18 locations tested. The model's overall accuracy is promising as it demonstrates that it is possible to model seasonal carbon fluctuations through two-dimensional models accurately. The mean squared error between observational measurements and the model are shown in Figure 2.

However, the model did have some inaccuracies. Specifically, there was disagreement between the modeled CO_2 variations and experimental data in equatorial regions. In regions such as equatorial Seychelles, the authors found that model fluctuations were 180 degrees out of sync; however, due to the lack of strong seasonality, the author's predictions for SCA were still accurate despite strong disagreements in seasonality dynamics between the model and experimental data. In view of this, Box and Gillette necessitate a need to model equatorial regions with a separate model to account for the lack of seasonality in the region. Furthermore, while the SCA values calculated remained accurate in most regions, the inclusion of certain assumptions leads to day-to-day inaccuracies. Significantly, the model assumes that the oceans are abiotic, leading to reduced modeled bioflux in latitude bands which include significant amounts of the ocean, partially contributing to some of the inaccuracies estimating biospheric fluxes in equatorial regions.

When considering the long-term utilization of the model, the model provides parameterization for future data by relying on experimental data for its flux calculations. However, this choice of parametrization limits how far out the model can predict in the first place; specifically, it is difficult to accurately predict parameters such as annual evapotranspiration rate and wind speed for time scales ten or more years into the future. The second factor which restricts the utility of the model in the future is the modeling of anthropogenic releases of carbon. The model used to predict fossil fuel emissions was published in a study from 1982; however, given the rapid increase in fossil fuel emissions over the past 40 years, it appears no longer possible to utilize the data values for fossil fuel flux in the model and still maintain accurate predictions. Therefore, for the model to be feasible today, it would be necessary to utilize more current data or utilize emissions scenarios such as those presented by the IPCC to have more accurate predictions.

The Box and Gillette model has provided promising results in modeling seasonal carbon amplitudes in the short term; however, when considering longer time scales such as predicting seasonal carbon amplitudes 80 to 100 years in the future; it is no longer feasible to model carbon dioxide using an approach driven heavily by experimental data. An alternative to the flux-based model introduced by Box and Gillette is Zhao et al.'s computational modeling approach, which considers global seasonal carbon amplitudes. The model examines historical data ranging from 1850-to 2005 and then utilizes future projections using various computational earth system models. Through this process, the authors can establish predictions for seasonal carbon fluctuations from the year 2090 to 2100. Furthermore, the model provides additional insights into the role of vegetation in dictating the carbon cycle.

The model provided by a study by Zhao et al. combines the results from 10 different Earth system models, which all have their own representations of the carbon cycle Zhao and Zeng 2014. The study's authors began their examination by applying curve-fitting procedures to historical carbon dioxide records. The algorithm first fits the function with a quadratic polynomial and then filters the result using a Fourier transformation accounting for seasonal fluctuations. The authors established a basic model for historical carbon fluctuations through this process, allowing them to examine the global carbon budget through a simple differential equation

$$\frac{\mathrm{dCO_2}}{\mathrm{d}t} = \mathrm{FFE} - \mathrm{NBP} + \mathrm{FOA} \tag{9}$$

Utilizing this model, the authors theorize that the change in the CO2 over time is equal to net Fossil Fuel Emissions (FFE) minus net biosphere production (NBP) plus net ocean-atmosphere flux (FOA). However, the authors theorized that changes due to FFE are largely replicated solely through net biosphere productivity over long time scales.

Utilizing the carbon cycle functionality of various climate models finds varying results over a long time scale. The author's examination of 10 earth climate systems concluded that all the earth cycle models tested found an increase in amplitude over time. However, there was considerable variance in the predicted amplitude over longer time scales. Furthermore, the cyclical dynamics predicted differed as some models shifted periods of maximum and minimum carbon levels to later times during the year. Zhao et al.; find the most accurate results through the use of the multi-model ensemble, where the amplitude trends are close to those observed at surface stations.

Through the use of earth system models, in addition to establishing long-term predictions for seasonal carbon amplitude, the authors also attempt to better understand how changes in net biosphere production are influenced by photosynthesis and respiration. The authors attempt to better understand the contributions of the two processes by computing the mean seasonal cycle of the detrended $\mathrm{CO_2}$ growth rate for past data from 1961 to 1970 and projections made from 2090 to 2100. The authors find that land activities are the main driver of $\mathrm{CO_2}$ amplitude differences. However, when examining the results for the two time periods, the variance between the different earth system models led to inconsistent increases in both NPP and $\mathrm{CO_2}$ levels from the past data to future projections. Thus the authors used the results of the muti-model ensemble, which showed a 62% increase in $\mathrm{CO_2}$ and a 68% increase in NBP (net biosphere production) (Zhao and Zeng 2014). Therefore, the authors conclude that the CO_2 amplitude increase is primarily driven by photosynthetic processes and less by respiration.

The earth climate systems projections to 2090 provide several other important insights into future changes in carbon amplitudes. Significantly, the authors find that there is a

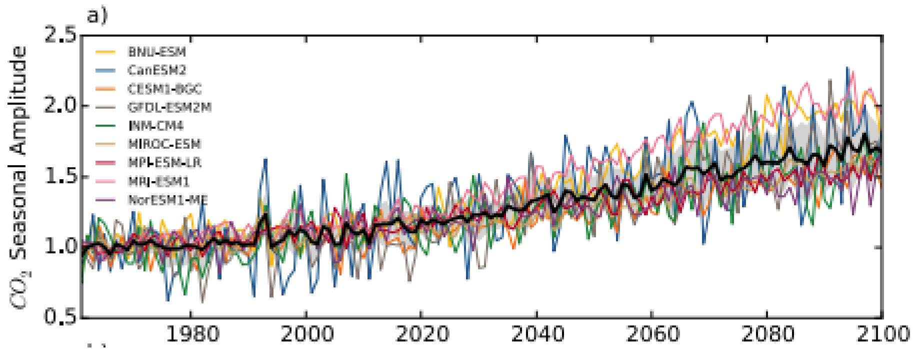

Figure 3: Divergence of Seasonal Amplitude Estimates for Earth-System Models

seasonal shift in northern latitudes as peak carbon intake shifted from July to June. The future overall increase in carbon amplitudes is concentrated in higher latitude regions. The authors computed zonal averages of carbon fluctuations for various latitudes and found that increases in carbon amplitude were dominated by regions north of 45 degrees N, where there was an increase in the length of the growing season due to climate-induced warming; on the other hand, changes in the northern subtropical region (10-30 degrees N) and in the Southern Hemisphere partially offset increases in the Northern latitudes, due to tropical ecosystems experiencing prolonged dry seasons due to climate change, thus reducing their seasonal carbon amplitudes.

An important consideration of the results published by Zhao et al. was the lack of agreement between the multi-modal approach and other earth system models, where most models predict an increase in seasonal amplitude at every latitude. The divergence of different earth-system models is shown in Figure 3. This disagreement between model dynamics was addressed, mostly due to the difficulty in predicting climate systems over such long time scales. Zhao et al. identified two major driving mechanisms for increases in seasonal carbon amplitude, carbon fertilization and increased greening in warmer climates. However, while these trends contribute to strong increases in seasonal carbon amplitude, the unpredictability of climate change could lead to drastically different results, which are represented in other models. Zhao et al. provide the example of soil moisture and surface temperature changes, where increased temperatures lead to less moisture in the aquifer which thus leads to reduced vegetation which would drive seasonal carbon amplitudes down, thus adding additional complexity to the dynamics of climate models. Thus, Zhao et al. necessitate the need for more comprehensive modeling parameters and the use of more consistent dynamics between models to yield more accurate results.

The examination of both the Box and Gillette schematic model and the results of the computational earth system model study of Zhao et al.; both support our original findings that changes in seasonal carbon amplitude are driven by vegetation. However, both studies reveal that predicting seasonal carbon amplitudes to high degrees of accuracy remains difficult. We find that reliance on observational data while providing more accurate short-term predictions for the Box and Gillette model causes difficulties when making long-term predictions. Additionally, while able to replicate many of the dynamics causing seasonal climate amplitudes in the first place, a computational approach does not lead to conclusive results due to variances in the dynamics of various climate system variables. The difficulties of modeling seasonal climate amplitudes become more evident when considering human-induced climate change, whose effects on the short-term carbon cycle are still unclear.

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